HISSAN CENTRAL EXAMINATION - 2079 (2022)

Grade: XII

F.M.: 75

Time : 3 hrs

COM. MATHEMATICS (0081)

Candidates are required to give their answers in their own words as far as practicable.

Attempt ALL Questions.

GROUP A

Rewrite the correct option in your answer sheet. [11 × 1 = 11]

1. One of the roots of the equation $z^2 + z + 1 = 0$ is

   a) $\frac{1+\sqrt{-3}}{2}$ b) $\frac{-1+\sqrt{-3}}{2}$ c) $\frac{-1-\sqrt{-3}}{2}$ d) $\frac{-1+\sqrt{3}}{2}$

2. If the equation $px^2 - 30x + 25 = 0$ have equal roots, then the value of p is

   a) 7 b) 8 c) 9 d) 10

3. If $\sin x = \sqrt{3}\cos x$, then the value of x is

   a) $n\pi - \frac{\pi}{6}$ b) $n\pi - \frac{\pi}{3}$ c) $n\pi + \frac{\pi}{3}$ d) $n\pi$

4. If $\tan 4x - \tan x = 0$, then the value of x is

   a) $n\pi + \frac{\pi}{3}$ b) $2n\pi + \frac{\pi}{3}$ c) $\frac{n\pi}{5}$ d) $\frac{n\pi}{3}$

5. $\vec{k} \times \vec{j} =$

   a) 1 b) 0 c) $-\vec{i}$ d) $\vec{i}$

6. The eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a < b$ is

   a) $\sqrt{1 - \frac{b^2}{a^2}}$ b) $\sqrt{\frac{b^2}{a^2} - 1}$

   c) $\sqrt{1 - \frac{a^2}{b^2}}$ d) $\sqrt{\frac{a^2}{b^2} - 1}$

7. If $A \subseteq B$ with $P(B) > 0$, then the value of $P(A|B)$ is

a) $\frac{P(A \cap B)}{P(B)}$ b) $\frac{P(B \cap A)}{P(A)}$ c) 1 d) 0

8. The point on the curve $y^2 = x$, the tangent at which makes an angle of 45° with the x-axis is

a) $\left(\frac{1}{4}, \frac{1}{2}\right)$ b) (-1, 0) c) (1, 2) d) (1, 3).

9. The differential equation whose solution is $x^2 + y^2 = C$ is

a) $x\,dy + y\,dx = 0$ b) $x^2\,dx + y^2\,dy = 0$

c) $x\,dx - y\,dy = 0$ d) $x\,dx + y\,dy = 0$

10. Given the system of equations $2x + y - 3z = -10$, $-2y + z = -2$, $z = 6$, the values of x, y, z are

a) 2, 4, 6 b) 2, 7, 6 c) 3, 4, 6 d) 2, 4, 5

11. If one of two like parallel forces and their resultant are 12 N and 28 N respectively, the other force is

a) 14 N b) 16 N c) 18 N d) 40 N

OR

Given the national income model $Y = E$; $E = C + I$, where $C = 280 + 0.6Y$, $I = 80$, the values of the intercept and slope of the expenditure equation are

a) 360, 0.6 b) 280, 0.4 c) 280, 0.6 d) 360, 0.4

GROUP B

[5 × 8 = 40]

12. a) State De Moivre's theorem. Use it to find cube roots of unity.

b) Form a quadratic equation whose roots are four times the roots of $6x^2 - 13x + 6 = 0$. [1+2+2]

13. a) Use mathematical induction to prove that for any positive integer n, $6^n - 1$ is multiple of 5.

b) Solve the system $x - y = 0$, $2x - y + 4z = 18$, $-3x + z + 2 = 0$ by Row–equivalent matrix method. [2+3]

14. a) If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$ prove that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$.

b) Find the eccentricity and foci of the hyperbola $3x^2 - 4y^2 = 36$. [3+2]

15. What is correlation? Write its types. Also coefficient of correlation between X and Y for 20 items is 0.3, mean of X is 15 and that of Y 20, standard deviations are 4 and 5 respectively. At the time of calculations one item 17 was wrongly copied instead of 27 in case of X- series and 35 instead of 30 in case of Y series. Find the correct coefficient of correlation.

[1+4]

16. Find the value of $\int \frac{\sin x \cos x}{(\sin x + \cos x)^2}\, dx$ [5]

17. Define differential equation. What are the methods of solving differential equation?

Solve the differential equations $x \ln x \frac{dy}{dx} + y = 2 \ln x$ [1+1+3]

18. A factory manufactures two products, A and B. Each product requires the use of two machines, Machine I and Machine II. The total hours available, respectively, on Machine I and Machine II per month are 180 and 300. The time requirements and profit per unit for each product are listed below.

	A	B
Machine I	1	2
Machine II	2	2
Profit	20	30

a) Formulate the given problem mathematically.

b) How many units of each product should be manufactured to maximize profit, and what is the maximum profit? Solve the problem by Simplex Method.

[1+4]

19. a) If the magnitudes of two unlike parallel forces P, Q $(P > Q)$ are increased by the same amount, show that the line of action of the resultant will move away from P.

b) From a point on the ground at a distance of 12 m from a vertical wall, a ball is thrown at an angle of 45°. It just clears the wall and strikes the ground at a distance of 6 m on the other side. Find the height of the wall.

[2+3]

OR

State the Hawkins - Simon conditions for the viability of the system.

Given the following input-output table, calculate the gross output so as to meet the final demand of 200 units of agriculture and 800 units of industry.

	Consumer Sector		
Producer Sector	**Industry**	**Agriculture**	**Final Demand**
Agriculture	300	600	100
industry	400	1200	400

[1+4]

GROUP C

[8 × 3 = 24]

20. a) How many numbers of 4 different digits can be formed from the digits 4, 5, 6, 7, 8. How many of these numbers are divisible by 5? How many of these are not divisible by 5?

b) If $C_0, C_1, C_2, \ldots, C_n$ are binomial coefficients in the expansion of $(1+x)^n$, prove that

$$C_0 - 2C_1 + 3C_2 - \ldots + (-1)^n (n+1) C_n = 0.$$

c) Define a group. Let $a * b = 3a + 2b$ for $a, b \in \mathbf{Z}$. Is $*$ associative? Justify your answer. [3+3+2]

21. a) If A, B, C are points (1, 0, 4), (0, −1, 5) and (2, −3, 1) respectively. Find the foot the perpendicular drawn from A to the line BC.

b) Using vector method, prove that
$\sin(A + B) = \sin A \cos B + \cos A \sin B$. (5+3)

22. Let $f(x) = \cos^{-1} x$. Find $\frac{d}{dx} f(x)$ from first principle. State Rolle's Theorem.

Verify the theorem for the function $f(x) = x(x-3)^2$ on $[0, 3]$. [4+1+3]

THE END